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A NEW SEPARATION AXIOM ON BITOPOLOGICAL SPACES

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1.1 Introduction

The notions of quasi-proximities and quasi-uniformities inevitably arise in literary works dealing with nonsymmetrical metrizable spaces. The idea of a bitopological space originated from Kelly's finding that two distinct topologies on a non-empty setX were produced by the asymmetric behavior of quasipseudo metric and its conjugate when studying nonsymmetrical spaces, which are also called quasi-metric spaces. Regarding this, Kelly's groundbreaking work [28] was an important step forward because it presented and explored the idea of a bitopological space. Consequently, quasi-metric, quasi-uniform, or quasi-proximity spaces are the original contexts for the idea of bitopological spaces. This provided a framework for investigating non- empty sets in relation to two distinct topologies. The phenomenon of bitopological spaces emerged from this foundational idea and has since attracted the attention of numerous modern topologists, who have contributed to the field's growth by developing new ideas like pairwise connectedness, pairwise compactness (and its invariants like pairwise Lindelöf), pairwise countable compactness, pairwise paracompactness, and several pairwise variants of other covering properties and separation axioms. We are aware that, on occasion, many other separation axioms have been proposed and investigated for topological and bitopological spaces. Urysohn provided the initial comprehensive analysis of separation axioms on topological spaces. Additionally, the separation axioms were discussed in greater depth by Van and Freundenthal. A new axiom of separation between T_0 and T_1 was proposed by Aull. Separation axioms for topological and bitopological spaces have been proposed by several mathematicians in recent years, including Ekici Arar Mukharjee Wianwiset et al., Khalaf et al. Rao, and Narasimhan Gowri and Rajayal. It was for this reason that we proposed a novel separation axiom for a bitopological space spanning the interval between the sets T_0 and T_1.

The term "bitopological space" will be used throughout this chapter to refer to a nonempty set X that has two non-identical topologies, $\Im_1$ and $\Im_2$. A subset of X's closure and interior are meaningful in a broad sense. The new pairwise T_D* axiom,which we introduce at the beginning of the chapter, is a separation axiom. Afterwards,we present a novel topology on a bitopological space $(X, \Im_1, \Im_2)$ as a topology that encompasses all subsets of X that are either closed in both topologies or open in both. This subset is then rendered closed in the new topology. We refer to the new topological space that is generated by the set X and its topology as $(X, \Im R)$. At last, the relationship between the separation axiom T_D* and the topology $\Im R$ is proven.

1.2 REVIEW OF LITERATURE

Mohsen, Salim. (2022). In this paper, we introduce a novel class of compact spaces and investigate their characteristics; specifically, we focus on the #RG-compact space in topological spaces onto which the # rg-open set is superimposed, while simultaneously revising some compact space theorems.

Bayhan, sadık (2021) While writing on intuitionistic topological space, the second author first brought the idea up. This paper's goal is to prove that category of intuitionistic topological spaces and continuous mappings and category of bitopological spaces and pairwise continuous mappings are natural functors. We derive certain links between these and previously specified ideas after applying these functors to generalize bitopological notions of separation to intuitionistic topological spaces.

Roy, B. & Noiri, T.. (2021). The study in this work focuses on γμ -open sets and γμ - closed sets in a GTS (X, μ), where γμ is an operation from μ to P(X). Typically, groups of γμ -open sets are less numerous than groups of μ-open sets. Here we also establish the conditions under which they are equal. The discussion has focused on a few characteristics of these sets. We also define and talk about the features of some closure-like operators. The connection between closure operators of comparable typeson the GTS (X, μ) has been proven. The novel closure-like operator is shown to be a Kuratowski closure operator under certain conditions. With the aid of this recently established closure operator, we have also created a special kind of closed sets called γμ-generalized closed sets and gone over some of its fundamental characteristics. We have presented a few weak separation axioms and gone over their characteristics as anexample of an application. At last, we have demonstrated a few preservation theoremsof these broad ideas.

Al-shami, Tareq. (2021). No soft topological space generated by a soft information system can be anything other than soft compact, as is well known. We present novel varieties of soft compactness on finite spaces and explore their applications in information systems by integrating soft compactness with partially ordered sets. We begin by introducing the concept of monotonic soft sets and defining its key characteristics. Secondly, we define ordered soft compact spaces and monotonic soft compact spaces and demonstrate their interrelationships through examples. We provide a detailed description of each of them by utilizing the finite intersection property. Additionally, we investigate certain features linked to finite product spaces and certain soft ordered spaces. Additionally, we explore the circumstances in which these ideas are maintained between the soft topological ordered space and its parametric topological ordered spaces. Finally, we present a method based on the ideaof ordered soft compact spaces that may anticipate the information system's objects' missing values.

1.3 A Separation Axiom Between Pairwise T0 and Pairwise T1

X is a non-empty set with two topologies, and we begin this section by introducing a new separation axiom on a bitopological space (X, ℑ1, ℑ2).

Definition 6.2.1 You can discover an open set Ui in ℑi that meets the condition: Ui -

{x} = P1 ∩ P2 where Pi ∈ ℑi and x ∉ P1 ∈ P2 for every i ∈ {1, 2}. This means that the bitopological space is pairwise TD∗

The following is a bitopological space (X, ℑ1, ℑ2) equivalent of the pairwise TD∗ separation axiom:

Definition 6.2.2 For each element x in X, there must be an open set Ui ∩ (ℑ1 cl{x} ∪

ℑ2 cl{x}) = {x} such that for every i in {1, 2} for a bitopological space (X, ℑ1, ℑ2) to be pairwise TD∗.

As can be seen below, both interpretations of the TD∗. pairwise axiom are equivalent.

(X, ℑ1, ℑ2) is a topological space that meets the requirement stated in Definition

6.1.1. If x is an element of X, then for every i in the set {1, 2}, there is a set Ui that contains x. We may express Ui - {x} as P1 ∩ P2, where Pi is an element of ℑi and x is an element of P1 that is not in P2.

Let y ∈ P1 ∩ P2. Consequently, P1 and P2 are open sets in ℑ1 and ℑ2 that contain y, respectively.

No Pi also does not contain x.

Pi ⋂ ℑi cl{x} = ϕ is our first assertion. The sentence can be paraphrased as: "Let z ∈

Pi and z ∈ ℑi cl {x}." A subset of ℑi that contains z also contains x.

Specifically, there is a contradiction (x ∈ Pi) for all i ∈ {1, 2}.Therefore, it follows that Pi ⋂ ℑi cl{x} = ϕ. It follows that Pi ⋂ ℑi cl{x} = ϕ ⟹ (P1 ⋂ P2) ⋂ (ℑi cl{x} ⋃ℑj cl{x}) = ϕ for all i ∈

{1, 2} ⟹ (P1 ⋂ P2) ⋂ (ℑi cl{x} ⋃ℑj cl{x}) = ϕ ⟹ Ui ∂ (ℑi cl{x} ⋃ℑj cl{x}) = {x}.

On the other hand, if Ui is a subset of the set consisting of functions cl{x} and cl{j}, then {x} is also a subset of Ui. Then, X - (Ui - {x}) = (X - Ui) ⋃ {x} = (X - Ui) ⋃ (Ui
⋂ (ℑi cl{x} ⋃ℑj cl{x})) Lastly, X - (Ui - {x}) = (X - Ui) ⋃ {x} = (X - Ui) ⋃ (Ui ⋂

(ℑi cl{x} ⋃ℑj cl{x})) = X ⋂ ((X – Ui) ⋃ (ℑi cl{x} ⋃ℑj cl{x})) = ((X – Ui) ⋃ (ℑicl{x} ⋃ℑj cl{x})) = ((X – Ui) ⋃ (ℑi cl{x}) ^ℑj cl{x} = F1 ⋃ F2 in the case where X
- Fi ∈ ℑi and x ∈ F1 ⋂ F2. Similarly, ⟹ Ui - {x} = P1 ⋂ P2 in the case where Pi ∈

ℑi and x ∉ P1 ⋃ P2.

The pairwise TD∗ axiom displays a number of distinctive features, including:

The 6.2.3 Theorem The equality T_1 ⟹ TD∗ ⟹ T_0 holds in a bitopological space (X,

ℑ1, ℑ2).

The evidence. The bitopological space (X, ℑ1, ℑ2) can be thought of as pairwise T_1. Every singleton in a paired T_1 bitopological space is closed in both topologies, meaning that for any x in X, ℑ1 cl{x} ⋂ℑ2 cl{x} = {x}. X - {x} is ℑ1 open and ℑ2 open, while Ui - {x} is ℑi open for every ℑi neighbourhoods of x in Ui. Define P1 as Ui - {x} and P2 as X - {x}. Therefore, according to the pairwise TD∗ axiom, Ui - {x}
= P1 ⋂ P2.

Presently, think of the bitopological space as (X, ℑ1, ℑ2) pairwise TD∗. Assume thatX has two unique elements, x and y. In order to demonstrate that (X, ℑ1, ℑ2) fulfills the pairwise T_0 axiom, we prove that both x and y have neighbourhoods in their respective topologies that do not contain each other. Based on the first postulate of pairwise TD∗, for any x ∈ {1, 2}, there is a neighborhood Ui of x such that Ui - {x} =P1 ⋂ P2, where each Pi is open in ℑi and none of them include x. We can conclude that y is a subset of Ui for all i in the set {1, 2}. P1 and P2 are ℑ1 and ℑ2
neighborhoods of y that do not contain x, respectively, if y is a subset of Ui for some i

∈ {1, 2}. Consequently, (X, ℑ1, ℑ2) is a pairwise T_0.

Conclusion 6.2.4 The resulting set of every element in a paired TD∗ bitopological space is closed in both topologies.

The evidence. The derived set of {x} in the topology ℑi ∀ i ∈ {1, 2} is denoted by ({x}i ′) in a pairwise TD∗ bitopological space (X, ℑ1, ℑ2).

In a bitopological space (X, ℑ1, ℑ2), we assert that if y is an element of ℑi cl({x}i ′), then y is either equal to x or an element of ({x}i ′). Every i neighborhood Ui of y overlaps {x}i ′ since y is in set cl({x}i ′). Given that p is a subset of {x}i ′ ⋂ U, and that Ui is the i-th neighborhood of y, we can deduce that Ui contains x and that y is either equal to x or a subset of ({x}i ′).

Next, we need to show that the set ℑi cl({x}i ′) does not contain x for every i in {1, 2}in order to establish that the derived set of {x} is closed in both topologies in a pairwise TD∗ bitopological space (X, ℑ1, ℑ2). There is an open neighborhood Ui of x in ℑi that meets the criterion, since the space (X, ℑ1, ℑ2) has been thought of as pairwise TD∗. Ui is equal to {x} if and only if (ℑi cl{x}) is equal to {x}, and Ui is equal to {x} if and only if ({x}i ′) is equal to ϕ.

The derived set of every element is closed in both topologies since x is not a member of ℑi cl({x}i ′).

The 6.2.5 Theorem For any pair of different elements x and y in a pairwise TD∗ bitopological space (X, ℑ1, ℑ2), there is an open set Pi in ℑi such that P1 ⋂ P2 contains either just x but not y or only y but not x, where i ≠ j and j is a member of {1,2}.

The evidence. With two unique elements x and y in the pairwise TD∗ bitopological space (X, ℑ1, ℑ2), there exists an i-th neighborhood Ui of x in X with i ∈ {1, 2} and the condition Ui - {x} = P1 ⋂ P2 holds, meaning that every Pi is open in ℑi and none of them contain x. Two situations have now emerged:
First case: x ∈ U1 ⋂ U2, but y ∉ U1 ⋂ U2 and we obtain the result if y ∉ Ui for every i in the set {1, 2}.

The second case is when y is a member of Ui for either i = 1 or i = 2, and according tothe pairwise TD∗ axiom, Ui - {x} = P1 ⋂ P2. Here, P1 and P2 are ℑ1 and ℑ2 neighbourhoods of y, respectively, that do not contain x.

1.4 New Topology Induced on a Bitopological Space

Every time a new class of closed or open sets is introduced, and topologies on those sets are subsequently introduced as well, the subject of topological spaces and bitopological spaces is enhanced. When novel topologies are introduced, new dimensions are opened up for the study of topological characteristics under special circumstances. To induce a new topology on a bitopological space, we define a new closure operator in this section.

Section 6.3.1 Definition Look at the bitopological space (X, ℑ1, ℑ2) as an example. The function R is defined on the interval (1, 2): The collection of all subsets of X is denoted as P{X}, where P{X} is defined as R (1, 2) (A) = {y ∈ X | ∀ U ∈ ℑ1 and V
∈ ℑ2 | y ∈ U - V, (U - V) ⋂ A ≠ ϕ.

Meaning 6.3.2 A bitopological space (X, ℑ1, ℑ2) also allows for the definition of the function R (1, 2) as:

The following demonstrates that the two interpretations of the R (1, 2) function are interchangeable:

Assume that for every x in {y in X | ∀ U ∈ ℑ1 and V ∈ ℑ2 |, where y is in U - V, the product of (U - V) and A does not equal ϕ.

So, in the ∀ℑ1 area, For every neighborhood X in ℑ2 and open set V that does not contain x, we get U ⋂ (X - V) ⋂ A ≠ ϕ. U of x and (∀ℑ2) closed sets F that include x, where U ⋂ F ⋂ A ≠ ϕ.

Specifically, for every x ∈ {y ∈ X | ∀ U ∈ ℑ1 | y ∈ U, ℑ2cl{y}⋂ A ⋂ U ≠ ϕ}, we have that U ⋂ A ⋂ ℑ2cl{y} ≠ ϕ.
For any x in the set {y ∈ X | ∀ U ∈ ℑ1 | y ∈ U, ℑ2cl{y} ⋂ A ⋂ U ≠ ϕ}, we have cl{x} ⋂ A ⋂ U ≠ ϕ ∀ 1 neighborhood. Union of x.

The intersection of all ℑ2 closed subsets of X that contain x is ℑ2 cl{x}, which is true for all ℑ1 neighborhood. The neighborhood of x and the closed set F that contains x for any n ∈ ℑ1 where (U ⋂ F) ⋂ A ≠ ϕ. The neighborhood of x in U and the open set V that contains x in ∀ ℑ1 is denoted by (U ⋂ (X - V)) ⋂ A ≠ ϕ. Given an open set V that contains x and a union U of x, where x ∈ ℑ1, we have that (U - V) ⋂ A ≠ ϕ. Then, for any x in the set {y in X | ∀ U in ℑ1 and V in ℑ2, where y is in U - V, we have that (U - V) ⋂ A ≠ ϕ.

Proposition 6.3.3 R(1, 2) is an operator that asserts the following:

(i) A belongs to the set R(1, 2), (A) is a subset of ℑ1 cl(A), and A is a subset of P(X)(ii)A, B ⊆ P(X) if and only if (iii) R(1, 2) (A ⋂ B) ⊆ R(1, 2) (A) ⋂ R(1, 2) (B) and
(iv) A ⌊ B ⟹ R(1, 2) (B) ⊆ R(1, 2) (A) ⌈ A, B ⊆ P(X)The set (v) R(1, 2) This meansthat the product of R(1, 2) and A is equal to R(1, 2) plus R(1, 2) plus B.

The evidence. (I) Assume that x is a member of U - V and that (U - V) ⋂ A = ϕ, then there exists a set U ⊈ R(1, 2) and a set V ◎ R(2,) that meet this condition.

If x ∈ A then x ∈ U ⋂ A whereas x ∉ V ⋂ A. ⟹ (U – V) ⋂ A ≠ ϕ which is a contradiction.

*x is an element of A.

Consequently, A is a subset of R(1, 2) that contains A.

Moreover, if x is a subset of ℑ1 cl(A), then there is an open set U ⊈ℑ1 that is both disjoint from A and contains x.

We obtain x ∈ U - V and (U - V) ⋂ A = ϕ by assuming that X - ℑ2 cl(U) = V ∈ℑ2. ⟹

x belongs to the set R(1, 2) in the set A.

Let x be a real number in the interval [1, 2] (A) and B be a subset of A. Under these circumstances, one can always identify U ∈ 1 and V ∈ 2 such that x ∈ U - V and (U
- V) ⋂ A = ϕ.
For every x ∈ R(1, 2), where A ⋂ B, the function ϕ(U - V) ⋂ A ⋂ B is defined.On the other hand, x belongs to R(1, 2) (B) and B ⊆ A.
From (ii), we may deduce that R(1, 2) (A ⋂ B) ⊆ R(1, 2) (A) and R(1, 2) (A ⋂ B) ⊆

R(1, 2) (B) since (A ⋂ B) ⊆ A and (A ⋂ B) ⊆ B, respectively. If A is a subset of B, then R(1, 2) (A) is a subset of R(1, 2) (B).
Then, according to (i), R(1, 2) (A) is equal to R(1, 2) (R(1, 2) (A)). In this case, let xbe a real number in the interval [1, 2] (A).

We can locate open sets U and V in ℑ1 and ℑ2 that meet the following condition: x isin U - V and (U - V) is not equal to ϕ in A.

We assert that ϕ is equal to (U - V) ⋂ R (1, 2) (A).

The set of real numbers between 1 and 2, denoted as (U - V), is not equal to ϕ.

Our assumption remains the same: X contains an element y belonging to U ⋂ R (1, 2)

(A) and y ∉ V ∩ R (1, 2) (A), without limiting our generality.

⟹ y ∈ U ∈ ℑ1, y ∈ R (1, 2) (A) and y ∉ V ∈ ℑ2.

It is contradictory because for any y in R(1, 2), there is an A such that (U - V) ∩ A ≠ϕ.

The set R(1,2)(R(1,2)(A)) is equal to the set R(1,2)(A).

In order to establish this aspect, we employ the second interpretation of R (1, 2) (A).A ⊆ (A ∪ B) and B ⊆ (A ∪ B) are equivalent.
According to (ii), ⟹ R (1, 2) (A) ∪ R (1, 2) (B) ⊆ R (1, 2) (A ∪ B), and R (1, 2) (B)

⊆ R (1, 2) (A ∪ B).

Allow x to be a member of R(1, 2) (A) and R(1, 2) (B).

The variables x and y are both contained in the set of real numbers (1, 2) in both Aand B.
U and V are subsets of ℑ1 that contain x and meet the given criteria. ϕ = 2cl{x} ∩ B

∩ V, where A is a subset of U.

With E being equal to U ∩ V, we can deduce that ϕ is equal to ℑ2 cl{x} for every x ∈

R(1, 2) where A ∪ B.

If A is a subset of B, then R(1, 2) (A) is a subset of R(1, 2) (B).So, R(1, 2) (A ∪ B) is equal to R(1, 2) (A) ∪ R(1, 2) (B).
The R(1, 2) operator meets all the requirements of Kuratowski's closure operator on a

bitopological space, as shown by the previous theorem. Because of this, we set R(1, 2)to the following definition:

Meaning 6.3.4 A set A is considered to be R(1, 2) closed in the bitopological space(X, ℑ1, ℑ2) if and only if R(1, 2)(A) = A.

It is easy to deduce from Theorem 6.3.3 (i) that every ℑ1 closed set is also a R(1, 2) closed set. As demonstrated below, though, even a ℑ2 open set is closed in R(1, 2), which is an intriguing fact to keep in mind.

Since (X - A) is a ℑ2 closed set, for any ∀ x ∈ (X - A), ℑ2 cl{x} ⊆ (X - A). A is closed in R(1, 2) if and only if ℑ2 cl{x} ∩ A ∩ U = ϕ ∀ U ∈ ℑ 1.
Another closure operator R(2, 1) can be defined in the same way by switching the values of 1 and 2. The following is a definition of the R(2, 1) operator for the purpose of clarity:

Definition 6.3.5 Take into account a bitopological space (X, ℑ1, ℑ2) and make a function R(2,1): R(2, 1) (A) = {y ∈ X | ∀ U ∈ ℑ2 and V ∈ ℑ1 | y ∈ U - V, (U - V) ∩ A ≠ ϕ } | ∀ A ⊆ X, where U and V are subsets of ℑ1.

The following is an alternative way to define the R(2, 1) operator:The following is an alternative way to define the R(2, 1) operator:
In both cases, the meaning of the R(2, 1) operator is the same, as is readily apparent.In addition, this operator

can be used to close a loop. A subset of a set X in a bitopological space (X, ℑ1, ℑ2) can be defined as the closure of R(2, 1) in the following way:

Meaning 6.3.7 R(2,1) closed means that A is contained within X in the bitopological space (X, ℑ1, ℑ2).

A subset A of X in the bitopological space (X, ℑ1, ℑ2) is obviously closed in R(2, 1) ifand only if A is either ℑ2 closed or ℑ1 open.

In addition, a new closure operator R in the bitopological space (X, ℑ1, ℑ2) isgenerated from the closure operators given above and exhibits some intriguing characteristics. Here is the definition of this new closure operator:

Definition 6.3.8 R closedness is defined in a bitopological space (X, ℑ1, ℑ2) as the condition that A ⊆ X is equal to R(1, 2)(A) or R(1, 2)(A).The set A is said to be R closed if for any x ∈ A, there exists a set U1, U2 ∈ ℑ1 1 and a set V1, V2 ∈ ℑ 2 that meet the following condition: x ∈ U1 - V1 and x ∈ V2 - U2 with (U1 - V1) ⋂ A = ϕ and (U2 - V2) ⋂ A = ϕ. Alternatively, A can be said to be R closed if for any x ∉ A, there exists a set U ∈ ℑ1 and a set V ∈ ℑ2 such that x ∈ U and x ∈ V and ∈ ℑ2 cl{x}⋂ A ⋂ U = ϕ = 2.

Theorem 6.3.9 states that cl{x} is ⋂ A ⋂ V. The following conditions are met by the R operator:

The evidence. (i) Assuming x is included in R(A), we may find U1, U2 in ℑ1 and V1, V2 in ℑ2 that meet the following condition: x is in U1 - V1 and x is in V2 - U2 with (U1 - V1) ⋂ A =ϕ. and (U2 - V2) ⋂ A = ϕ. Since x is in both U1 and V1 subsets of A, we conclude that (U1 - V1) subsets of A do not contain , and hence that x is a member of A. Thereby, A is a subset of R(A).

Now, for any i in the interval [1, 2], there is a neighborhood of x denoted by Ui in the interval [1, 2] such that Ui ⋂ A = ϕ. We obtain x ∉ R(A) by taking X - ℑj cl(Ui) = Uj ∈ ℑj, | j ∈ {1, 2} | i ≠ j.

(ii) Assume that B is a subset of A and that x is a member of R(A). Then, for every U1, U2 ∈ ℑ1 and every V1, V2 ∈ ℑ2, there is an element x such that x is in U1 - V1 and in V2 - U2, and for every (U1 - V1) ⋂ A = ϕ and (U2 - V2) ⋂ A = ϕ.

⟹ (U1 - V1) ⋂ B = ϕ and (U2 - V2) ⋂ A ⋂ B = ϕ as B is contained in A. Similarly,(U2 - V2) ⋂ B = ϕ.

The set R(B) is empty of x.

(iii) According to (ii), R(A ⋂ B) ⊆ R(A) and R(A ⋂ B) ⊆ R(B). Since (A ⋂ B) ⊆ A and (A ⋂ B) ⊆ B, we can deduce that R(A ⋂ B) ⊆ R(A) ⋂ R(B). (iv) R(A) ⊆ R(R(A)) is clearly shown by (i). For every x ∉R(A), there exists a set U1, U2∈ℑ1 anda set V1, V2 ∈ ℑ2 that meet the following condition: x ∈ U1 - V1 and x ∈ V2 - U2 such that (U1 - V1) ⋂ A = ϕ and (U2 - V2) ⋂ A =ϕ.

If (U1 - V1) ⋂ R(A) =ϕ, then (U2 - V2) ⋂ R(A) = ϕ, too, according to our assertion.

(U1 - V1) ⋂ R(A) ≠ ϕ and (U2 - V2) ⋂ R(A) ≠ ϕ are elements of X such that y ∈ U1

⋂ R(A) but y ∉ V1 ⋂ R(A) y ∈ U1 ∈ ℑ 1, y ∈ R(A) and y ∉ V1 ∈ ℑ 2 are elements of X.

However, there is a contradiction because (U1 - V1) ⋂ A ≠ ϕ, since y is a real numberand belongs to R(A).

⟹ The product of R and R(A) is equal to R(A).

Similarly, we can demonstrate that the set A is not empty and that (U2 - V2) is a subset of R(R(A)) with respect to x and R(A) is a subset of R(R(A)) for all v. We apply the second definition of R(A) to show this section.

R(A) ∪ R(B) ⊆ R(A ∪ B) is evident from (ii) onwards.

On the other hand, consider x as belonging to R(A) ∪ R(B). If U, P ∈ ℑ1 and V, Q ∈

ℑ2 include x, then ℑ2 cl{x}∩ A ∩ U = ϕ = ℑ1 cl{x}∩ A ∩ V and ℑ2 cl{x∩ B ∩ P =

ϕ = ℑ 1 cl{x}∩ B ∩ P.

Take into consideration E as U ∩ P and F as V ∩ Q.
Think about E as U ⊂ P and F as V ∩ Q, where x ∉ R(A ∪ B).A ∪ B ⟹ R(A) ∪ R(B) is invertible.
A product of R(A) and R(B) is equal to R(A) plus R(B).

Consequently, a topology on (X, ℑ1, ℑ2) is defined by the R operator since it meets allthe requirements of Kuratowski′s closure operator. The relevant topological space is (X ,ℑR), and we represent this topology as ℑR. ℑR is defined as follows:

Understanding 6.3.10 Think about the bitopological space (X, ℑ1, ℑ2) and the set A that is a subset of X. If A is a subset of R and for every x in A there is aℑ2-open set Uthat contains x and a ℑ2-open set V that contains x that meet the following condition: x ∈ (ℑ2 cl{x} ∩ U) ∪ (ℑ1 cl{x} ∩ V) ⊆ A, then A is a subset of R.

In addition, the topologies defined by operators R(1, 2) and R(2, 1) on a bitopological space (X, ℑ1, ℑ2) are finer than, and equal to, ℑ1 and ℑ2, respectively, since they both meet the requirements of Kuratowski′s closure operator.

The following conclusions are also derived from the preceding discussion:

For every bitopological space (X, ℑ1, ℑ2), let A be a subset of X. Next, in the event that;

- If A is open in ℑ1, then it is also open in R(1, 2) and closed in R(2, 1).
- If A is open on ℑ2 and closed on R(1,2), then A is also open on R(2, 1).
- A is open in R(2,1) and closed in R(1,2) if and only if A is ℑ1 closed.
- A is open in R(1, 2) and closed in R(2, 1) if and only if A is ℑ2 closed.

A subset A of X in a bitopological space (X, ℑ1, ℑ2) is considered open and closed in

ℑR if it is open in both topologies or closed in both ℑ1 and ℑ2. This conclusion is supported by the results given above.

First, think about the following set of subsets of X:

The set B(1, 2) is defined as the identity matrix {U - V | U ∈ ℑ2 and V ∈ℑ1}.

The function B(2, 1) is defined as the identity matrix {U - V | U ∈ ℑ1 and V ∈ ℑ2}.

Assuming U is in $\Im 1$ and V is in $\Im 2$, the equation B is equal to (U - V) multiplied by (V- U).

Then, the topologies produced by the R(2,1), R(2,1), and B operators, respectively, have B(1, 2), B(2,1), and B as their bases.

Case in point 6.3.10 An assortment of positive integers can be represented by the letter P. With $m \geq n$, let An be the set of all elements in X. Define $\Im 1$ as the set of all analytic functions on X where $n \geq 1$ and $\Im 2$ as the set of all discrete topologies on X.A discrete topology also applies to (P, $\Im$R).

If R cl(A) = X, then we say that set $A \subseteq X$ is R-dense (Definition 6.3.12). Here, X is referred to as the R-hull of A.

Proposition 6.3.13. In a bitopological space (X, $\Im 1$, $\Im 2$), if a set $A \subseteq X$ is R-dense,then for every i and j $\in \{1, 2\}$, we have that $(Ui - Vj) \cap A \neq \phi$ or $(Vj - Ui) \cap A \neq \phi$.

The evidence. With $U \in R$ and $V \in R$, we can write (U - V) as (V - U).A overlaps with every R-open set because it is R-dense.
$A \cap ((U - V) \neq \phi$ or $A \cap (V- U)) \neq \phi$ is an example of this.

Purpose of 6.3.14 In a bitopological space (X, $\Im 1$, $\Im 2$), a bi open filter F is a set ofnon-empty subsets of X that have the given characteristics.

(i) For every i in the set {1, 2}, F is greater than or equal to $\Im 1$ and F is lessthan or not equal to $\Im$i.

(ii) For any i from 1 to 2, there exists an element F in the set F such that $E \cap F$

$\in \Im$ i.

(iii) $G \in F$ and $G \subseteq H$ with $G, H \in \Im i \Longrightarrow H \in F \; \forall \, i \in \{1, 2\}$

Proposition 6.3.15 When two distinct elements in X have distinct biopen filters in Y, we say that the space Y is dense in X and that X is a pairwise T0 bitopological space (X, $\Im 1$, $\Im 2$).

Proposition 6.3.15 When two distinct elements in X have distinct biopen filters in Y, we say that the space Y is dense in X and that X is a pairwise T0 bitopological space (X, $\Im 1$, $\Im 2$).

Assumption 6.3.16 The induced topological space (X, $\Im$R) is always T1 if (X, $\Im 1$,$\Im 2$) is a pairwise T0 bitopological space.

The evidence. Consider two distinct elements p and q of the set X. It is enough to prove that $\Im$R cl{p} = {p} for all p in X in order to establish that (X, $\Im$R) is a topological space, as a space is considered to be T1 if and only if the closure of any singleton with respect to its topology is the singleton itself. The theorem is proved by contradiction, meaning that the bitopological space (X, $\Im 1$, $\Im 2$) is not pairwise T0 if the induced topology is not T1. Assume that q is an element of X distinct from p, and that it belongs to the set $\Im$R cl{p}.

$\Rightarrow \Im_2 cl\{q\} \cap \{p\} \cap U_q \neq \phi \ \forall \ \Im_1$ neighbourhood U_q of q and $\Im_1 \ cl\{q\} \cap \{p\} \cap V_q \neq \phi \ \forall \ \Im_2$ neighbourhood V_q of q.

$\Rightarrow \Im_2 cl\{q\} \cap \{p\} \neq \phi$ and $\Im_1 cl\{q\} \cap \{p\} \neq \phi$.

$\Rightarrow p \in \Im_2 cl\{q\}$ and $p \in \Im_1 cl\{q\}$.

$\Rightarrow$ every $\Im_1$ neighbourhood and every $\Im_2$ neighbourhood of p contains q.

Additionally, for every q, every r neighbourhood contains p, which contradicts the statement that (X, ℑ1, ℑ2) is pairwise T0. This is because {p} ⋂ Uq ≠ ∀ℑ1 neighborhood Uq and {p} ⋂ Vq ≠ ∀ℑ2 neighbourhood of Vq of q.

The result is that the induced topological space (X, ℑR) is T1 if the bitopological space (Xℑ1, ℑ2) is pairwise T0.

1.5 Relation Between TD∗ and ℑR

Presumption 6.4.1 If the pairwise TD∗ of two bitopological spaces (X, ℑ1, ℑ2) holds, then the topological space (X, ℑ R) is discrete.

The evidence. Assume that x is a subset of X and that the bitopological space (X, ℑ1,

ℑ2) is pairwise TD∗. It is sufficient to demonstrate that every set (X - {x}) or every singleton {x} is open in ℑR in order to prove that (X, ℑR) is discrete.

Every time there is a neighborhood Ui of y and a neighbourhood Uj of y, the definition of R cl(X - {x}) is {y ∈ X | ℑi cl{y} ⋂ (X - {x}) ⋂ Uj ≠ ϕ and ℑ j cl{y} ⋂ (X - {x}) ⋂ Ui ≠ ϕ }.

If x is in the set R cl(X - {x}), then for every i neighbourhood Ui of x and every j neighbourhood Uj of x, ℑi cl{x} ⋂ (X - {x}) ⋂ Uj ≠ ϕ and ℑ j cl{x} ⋂ (X - {x}) ⋂ Ui ≠ ϕ. Assume that z is an element of ℑi cl{x}, z is not equal to x, and z is a member of Uj, and that z is a subset of (X - {x}) ⋂ Uj.

X being pairwise TD∗ means that x has a neighborhood Uj where P1 ⋂ P2 and no Piin ℑi contains x, as Uj - {x}= P1 ⋂ P2.

"Pi" is a neighborhood of z that does not contain x if z is a member of Uj.z is not equal to ℩i times cl{x}. Either {x} is an open set in ℑR or the set X - {x} is closed in ℑR. The set (X, ℑR) is discrete because every singleton is open in ℑR.
The 6.4.2 Theorem If the set (X, R) is discrete and all derivative sets are closed inboth topologies in a bitopological space (X, ℑ1, ℑ2), then (X, ℑ1, ℑ2) is pairwise TD∗.

The evidence. Every singleton {x} in X is R-closed if the induced topological space(X, ℑR) in a bitopological space (X, ℑ1, ℑ2) is discrete.

(X - {x}) is equal to R cl(X - {x}).

⟹ For any i ≠ j in the set {1,2}, there exists a neighborhood Ui of x such that ℑjcl{x} ⋂ (X - {x}). Therefore, Ui equals ϕ.

Where Ui = {x} and ⟹ℑj cl{x} are not equal.It is also closed under the provided condition.

A neighborhood exists for x. In ℑi, vi such that {x}i ′ Thus, vi is equal to ϕ.If ℑi cl{x} ∩ Vi = {x}, then x has a neighborhood Vi in ℑi.
We obtain Wi ∩ℑi cl{x} ∪ℑj cl{x}) = {x} by assuming that Ui ∩ Vi = Wi.

The function X - (Wi - {x}) equals (X - Wi). (X - Wi) is equal to {x}. ** (ℑi cl{x}) The equation (ℑj cl{x}) is equal to ((X - Wi) ∪ℑi cl{x}) ∪ℑj cl{x}).

where Fi is equal to (X - Wi), and ⟹ X - (Wi - {x}) = Fi ∪ Fj. The functions Fj and

ℑi cl{x} are defined for every i and j ∈ {1,2}, where i is not equal to j.

The output is (X - (Fi ∊ Fj)) = (X - Fi) when the inequality (⟹ Wi - {x}) is considered. For every i, j ∈ {1, 2}, and all values of i ≠ j, the absolute value of X minus Fj is equal to Pi times Pj.

Pairwise TD∗ is ⟹ (X, ℑ1, ℑ2).

Proposition 6.4.3 (X, ℑ1, ℑ2) is pairwise TD∗ in a pairwise regular bitopological space(X, ℑ1, ℑ2) if (X, ℑR) is discrete.

Obejectives of the study:-

1. To generalized semi-open pre-open sets, g-open sets tec and theirbitopolocical analogs.
2. To construct new separation axioms in topological and bitoplogical space.

1.6 Conclusion

Objective 1. To generalized semi-open pre-open sets, g-open sets tec and their bitopolocical analogs.

Exploring generalized semi-open sets, pre-open sets, and their bitopological counterparts will be the first leg of this inquiry. Determining the intricate network of mathematical patterns that transcend the boundaries of the ordinary is the objective of this inquiry. Investigated here is the concept of g-open sets, a generalization that goes beyond classical openness to allow a more nuanced comprehension of spatial interactions. The emphasis moves to the comparable structures that might be present in dual space when we broaden our study to encompass bitopology. This exploration of generalized semi-open and pre-open sets explores uncharted terrain in an effort to understand the intricate patterns that emerge when traditional ideas are transformed and expanded to fit the dynamic nature of mathematical abstraction. There will be a lot more chapters in this quest. In addition to deepening our understanding of the theory underlying these concepts, our goal in conducting this inquiry is to uncover their real-world implications and applications in the context of mathematical analysis and problem-solving.

Objective 2. To construct new separation axioms in topological and bitoplogical space.
Pursuing novel separation axioms that expand mathematical topology's horizons is thegoal of this study. It accomplishes this by venturing into topological and bitopological space, the uncharted territory of inventiveness. To get there, we'll have to figure out how to differentiate between locations inside these spaces using new standards. The intricate structure of spatial relationships can be better understood with the help of

new insights made possible by this. As part of this research, which requires analytical and creative thinking, we will rethink and expand upon traditional notions to create a more sophisticated system for classifying places. Creating these novel separation axioms enhances the subject's theoretical attractiveness and may lead to useful applications. The foundational structures of many mathematical landscapes can be explored and understood through the prism of these axioms. An expansion of topology's theoretical foundations and new approaches to solving practical problems within the broader context of mathematical analysis are both anticipated outcomes of this effort. In order to achieve this, we will venture into uncharted territory.

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